EXHIBIT Q

TO RULE 4.2 STATEMENT OF DR. DOUGHERTY

LINEAR CAPACITANCE OF PLANAR CAPACITOR BASED ON FERROELECTRIC PLATE

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One reason for using film ferroelectric materials in microwave engineering is that they can be used for realizing nonlinear planar capacitors, representing a layered dielectric structure. The author showed earlier [1] that a given coefficient of capacitance modulation can be obtained by suitably fixing the form factor $g = h/d$, where h is the film thickness, d the gap width between the planar electrodes.

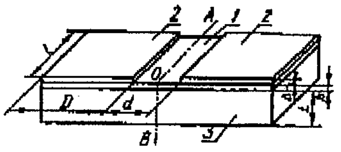

Fig. 1. Construction of planar capacitor: 1) ferroelectric film, 2) electrodes, 3) base.

In the present article we calculate the total linear capacitance C_l of a ferroelectric capacitor in the light of the influence of the scattering field in the dielectric base, and we give experimental data on the dependence of C_l on g.

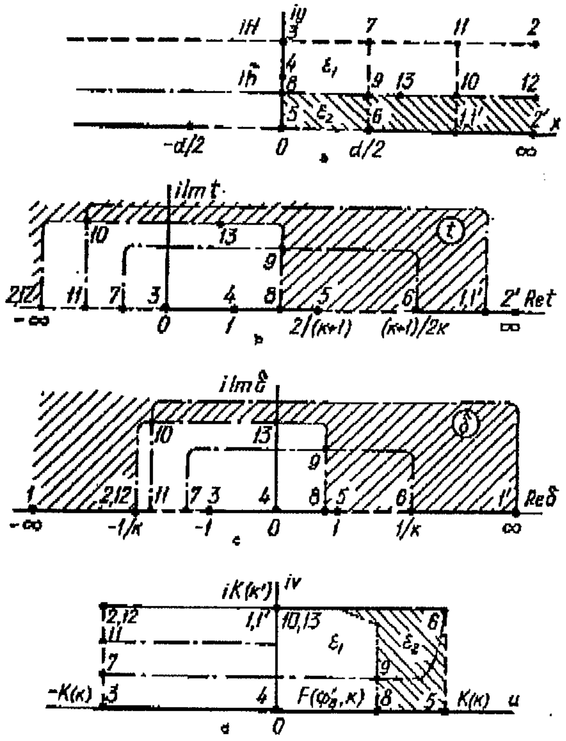

Fig. 2. Cross-section of two-layer corner capacitor (a) in z plane; intermediate stages of transformation (b, c); and its mapping into the w plane as a shunt connection of two plane capacitors (d).

To calculate the linear capacitance of the two-layer planar capacitor we use the method of conformal mapping in the approximation of linear and isotropic fillings. Using

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the mirror symmetry of the capacitor (Fig. 1) relative to the plane AOB through the gap center, we take as the theoretical model a corner capacitor whose cross-section is shown in Fig. 2a. It was shown in [1] that the function

$$dw/dz = -\pi[H\sqrt{2k}\,\mathrm{ch}\,(\pi z/H) - 1 - k^2]^{-1}$$

maps the half-strip of the z = x + iy plane (Fig. 2a) into a rectangle in the w = u + iv plane (Fig. 2d). Here, k = exp(-π/d2H) is the basic modulus of the complete elliptic integral of the 1st kind K(k), and H = h + L is the over-all thickness of the film h plus base L. The intermediate stages of the transformation of the corner into a plane capacitor are shown in Fig. 2b, c. The calculated coordinates of the main corresponding points (1-6) are given in Table 1.

Table 1

Point no.	Coordinates of points in plane			
	$z = x + iy$	$t = \mathrm{Re}\,t + i\,\mathrm{Im}\,t$	$\delta = \mathrm{Re}\,\delta + i\,\mathrm{Im}\,\delta$	$w = u + iv$
1	2	3	4	5
1,1′	$\frac{2H}{\pi}\,\mathrm{arth}\times\sqrt{1-\frac{2k}{k+1}} = x_1$	$1/k$	$-\infty, \infty$	$iK(k')$
2,2′	$\infty + iH, \infty$	$-\infty, \infty$	$-1/k$	$-K(k)+iK(k')$
3	iH	0	-1	$-K(k)$
4	$\frac{2H}{\pi}\,\mathrm{arth}\sqrt{1-\frac{2}{k+1}}$	1	0	0
5	0	$2/(k+1)$	1	$K(k)$
6	$d/2$	$(k+1)/2k$	$1/k$	$K(k)+iK(k')$
7	$d/2+iH$	$-\frac{2\,\mathrm{sh}^2(\pi d/4H)}{1+k}$	$-1-\pi d/6H$	$F(\arcsin\delta_7, k)$
8	ih	$\frac{2\cos^2(\pi h/2H)}{1+k}$	$1-2\pi h^2/dH$	$F(\arcsin\delta_8, k)$
9	$d/2+ih$	$1+\frac{\pi d}{4H}+i\frac{\pi dh}{4H^2}$	$1+\frac{\pi d}{2H}+i\frac{2\pi h}{H}$	$F(\arcsin\delta_9, k)$
10	x_1+ih	$1+\frac{\pi d}{2H}+i\sqrt{\frac{\pi^2hd^2}{4H^3}}$	$-1-\frac{d}{2h}+i\sqrt{\frac{Hd}{\pi h^2}}$	$-iK(k')$
11	x_1+iH	$\mathrm{Re}\,t_{11}$	$\mathrm{Re}\,\delta_{11}$	$-K(k)+iv_{11}$
12	$\infty+iH$	$\mathrm{Re}\,t_{12}$	$\mathrm{Re}\,\delta_{12}$	$\infty - K(k)+iK(k')$
13	x_1+ih	$\mathrm{Re}\,t_{13}+i\,\mathrm{Im}\,t_{13}$	$i\,\mathrm{Im}\,\delta_{13}$	$-iK(k')$

To find the position of the interface between the regions characterized by relative dielectric constants ε_1 and ε_2 in the w plane, the coordinates of the supplementary corresponding points (7-13) are calculated. The ratios of the dimensions of actual ferroelectric planar capacitors usually satisfy the inequalities

$$h/H \ll 1; \quad d/H \ll 1,$$

so that the coordinates of the supplementary corresponding points can be approximated; these results are also given in Table 1. It was assumed that g is upper-bounded by the ratio

$$g < H/\pi h.$$

Analysis of the supplementary point coordinates in the w plane shows that the interface is in this approximation perpendicular at point 8 to the lower electrode, and is characterized by maximum slope close to the upper electrode of the plane structure, while in essence it abuts onto the upper electrode at its center. This enables the planar capacitor with layer filling to be regarded as a shunt connection of two planar capacitors, with capacitance equal to the capacitance of the corner capacitor (per unit length)

$$\varepsilon_0\varepsilon_1 \frac{K(k)+F(\varphi_8, k)}{K(k')} + \varepsilon_0\varepsilon_2 \frac{K(k)-F(\varphi_8, k)}{K(k')},$$

equal to half the linear capacitance of the planar structure. Then, the total capacitance per unit length of the planar capacitor is given by

$$C_l = \frac{\varepsilon_0\varepsilon_1}{2K(k')} \{[K(k)+F(\varphi_8, k)] + \frac{\varepsilon_2}{\varepsilon_1}[K(k)-F(\varphi_8, k)]\}, \qquad (1)$$

where $\varepsilon_0 = (1/36\pi)\cdot 10^{-9}$ F/m; ε_1 and ε_2 are the relative dielectric constants of the base and film respectively, $k' = \sqrt{1-k^2}$ is the supplementary modulus of K(k), $F(\varphi_8, k)$ is the incomplete elliptic integral of the 1st kind, and $\varphi_8 = \arcsin(1 - 2\pi h^2/dH)$. The contribution of the value ε_2 of the film to the capacitance can be expressed with the aid of the ratio C_{lf}/C_l, where $C_{lf} = \varepsilon_0\varepsilon_2[K(k) - F(\varphi_8, k)]/K(k')$. In Fig. 3 we plot theoretical C_l and C_{lf}/C_l against g and the ratio $\varepsilon_2/\varepsilon_1$.

It should be mentioned that, on passing to the limit as h → H, expression (1) transforms to $C_l = \varepsilon_0\varepsilon_2 K(k)/K(k')$.

To confirm the theory and estimate its accuracy, we measured the capacitance of a two-layer device. The film part of the device was an 0.5 mm thick plate of VK-7 varicap ceramic, and it was cemented to a massive base of "polykor" (ε_1 = 10). The copper plane electrodes were obtained by evaporation in vacuo. To vary the form factor of the planar capacitor, the gap width was varied from 0.05 to 1 mm. The remaining dimensions were: l = = 10, D = 5, and L = 10 mm. The capacitance was measured by the interelectrode capacitance meter type E8-1 with relative error 1%. The value of ε_2 of the VK-7 ceramic was found directly for the plate employed in the device by the plane precision capacitor method and amounted to 2150 ± 50 at temperature T = 300 K. It can be seen from Fig. 4 that the theoretical and experimental results for C_l are within 5% in the range of g from 0.5 to 10.

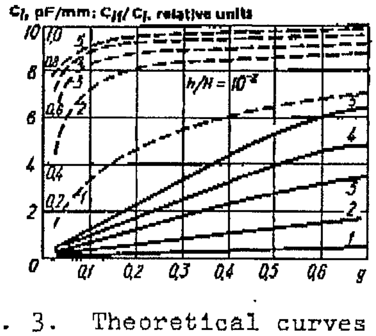

Fig. 3. Theoretical curves of capacitance C_l per unit length of planar capacitor and ratio C_{lf}/C_l (dashed lines) against form factor g with $\varepsilon_2/\varepsilon_1$ = 10 (1), 50 (2), 100 (3), 150 (4), 200 (5).

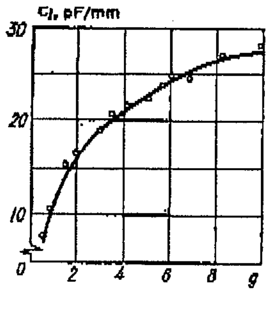

Fig. 4. Curve of C_l against form factor g (continuous curve plotted from (1)); f = 465 kHz; ε_1 = 10; T = 300 K; ε_2 = = 2150.

To sum up, the experimental data show that our method for calculating the capacitance per unit length enables fairly accurate account to be taken of the influence of scattering in the dielectric base. In particular, for g ≥ 0.7, the contribution of the parasitic capacitance is not more than 8% of C_l and falls as the form factor increases.

REFERENCE

1. A. G. Lipchinskii, "Effective modulation coefficient of a nonlinear capacitor in the context of its thermal properties," Izv. vuzov, Radioelektronika, vol. 17, no. 9, p. 43, 1974.

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